Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number ☐☐☐☐☐

Candidate Number ☐☐☐☐

Pearson Edexcel Level 3 GCE

Monday 18 October 2021 – Afternoon

Paper reference **9MA0/32**

Mathematics

Advanced

PAPER 32: Mechanics

You must have:
Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Unless otherwise indicated, whenever a value of g is required, take $g = 9.8$ m s^{-2} and give your answer to either 2 significant figures or 3 significant figures.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- The total mark for this part of the examination is 50. There are 5 questions.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

Pearson

1. A particle P moves with constant acceleration $(2\mathbf{i} - 3\mathbf{j})\,\text{m s}^{-2}$

At time $t = 0$, P is moving with velocity $4\mathbf{i}\,\text{m s}^{-1}$

(a) Find the velocity of P at time $t = 2$ seconds.

(2)

At time $t = 0$, the position vector of P relative to a fixed origin O is $(\mathbf{i} + \mathbf{j})\,\text{m}$.

(b) Find the position vector of P relative to O at time $t = 3$ seconds.

(2)

There is constant acceleration, therefore we can use SUVAT equations

a)

$S = ?$

$u = 4i$

$\boxed{v = ?}$

$a = (2i - 3j)$

$t = 2$

We need to find velocity v at t = 2:

$v = u + at$

$= 4i + (2i - 3j) \times 2$

$= 4i + 4i - 6j$

$= 8i - 6j$

b)

$\boxed{S = ?}$

$u = 4i$

$v = ?$

$a = (2i - 3j)$

$t = 3$

We need to find dispplacement S at t = 3:

$S = ut + \frac{1}{2}at^2$

$= (4i) \times 3 + \frac{1}{2}(2i - 3j) \times 3^2$

$= 12i + \frac{1}{2}(2i - 3j) \times 9$

Question 1 continued

$$= 12i + \frac{18}{2}i - \frac{27}{2}j$$

$$= 12i + 9i - \frac{27}{2}j$$

$$= 21i - \frac{27}{2}j$$

Given, position vector at $t = 0$ is $(i + j)$

$\therefore$ position vector at 3 seconds = displacement + $(i + j)$

$$= (21i - \frac{27}{2}j) + (i + j)$$

$$= 22i - \frac{25}{2}j$$

(Total for Question 1 is 4 marks)

2.

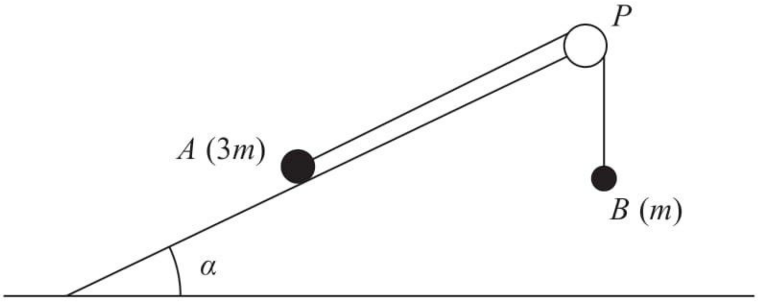


Figure 1

A small stone A of mass $3m$ is attached to one end of a string.

A small stone B of mass m is attached to the other end of the string.

Initially A is held at rest on a fixed rough plane.

The plane is inclined to the horizontal at an angle α, where $\tan\alpha = \frac{3}{4}$

The string passes over a pulley P that is fixed at the top of the plane.

The part of the string from A to P is parallel to a line of greatest slope of the plane.

Stone B hangs freely below P, as shown in Figure 1.

The coefficient of friction between A and the plane is $\frac{1}{6}$

Stone A is released from rest and begins to move down the plane.

The stones are modelled as particles.

The pulley is modelled as being small and smooth.

The string is modelled as being light and inextensible.

Using the model for the motion of the system before B reaches the pulley,

(a) write down an equation of motion for A

(2)

(b) show that the acceleration of A is $\frac{1}{10}g$

(7)

(c) sketch a velocity-time graph for the motion of B, from the instant when A is released from rest to the instant just before B reaches the pulley, explaining your answer.

(2)

In reality, the string is not light.

(d) State how this would affect the working in part (b).

(1)

Firstly, we need to highlight and note all important points in the question:

$mass_A = 3m \quad \therefore W_A = 3mg$

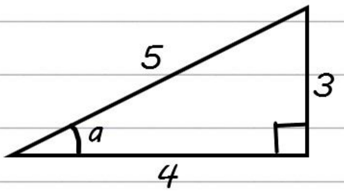
$mass_B = m \quad \therefore W_B = mg$

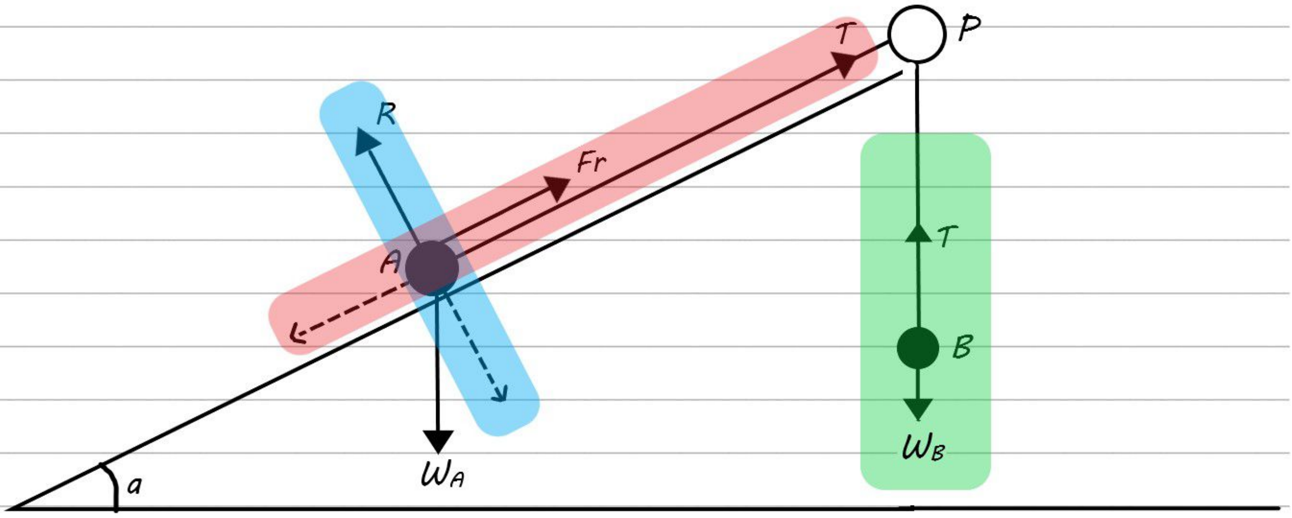
rough plane => friction = μR, where μ is $\frac{1}{6}$

$\tan a = \frac{3}{4}$

$\therefore \sin a = \frac{3}{5}$

$\therefore \cos a = \frac{4}{5}$





a) Since we know the net force acting on A as well as its mass, we can use Newton's Second Law:

$$F = ma$$

Parallel component of weight - Friction - Tension = ma

$W_A \times \sin a - Fr - T = 3ma$

$3mg \times \frac{3}{5} - \mu R - T = 3ma$

Question 2 continued

b) Resolving B vertically using Newton's Second Law:

acceleration of B is the same as acceleration of A,

$$T - W_B = ma$$

$$T = ma + W_B$$

$$\boxed{T = ma + mg}$$

Resolving A perpendicular to plane:

Since A is in equilibrium perpendicularly,

upwards force = downwards force

$$R = W_A \cos a$$

$$= \boxed{3mg \times \frac{4}{5}}$$

Substituting values in equation of motion for A from part (a):

$$3mg \times \frac{3}{5} - \mu R - T = 3ma$$

$$\frac{9mg}{5} - \frac{1}{6} \times 3mg \times \frac{4}{5} - (ma + mg) = 3ma$$

$$\frac{9mg}{5} - \frac{2mg}{5} - (ma + mg) = 3ma$$

$$\frac{7mg}{5} - ma - mg = 3ma$$

$$\frac{2\cancel{m}g}{5} = 4\cancel{m}a$$

$$a = \frac{1}{10} g$$

Q. E. D.

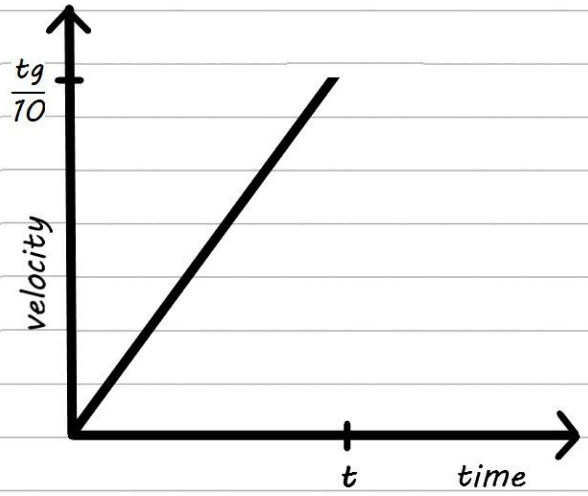
Question 2 continued

c) For B, the acceleration is the same as the acceleration for A

acceleration = $\frac{1}{10}g$

$u = 0$

∴ velocity is $\frac{t}{10}g$, where t is the time taken by B to reach the pulley



d) The tension will not be equal across all parts of the string, and thus different values of T would be needed when resolving A and B

(Total for Question 2 is 12 marks)

3.

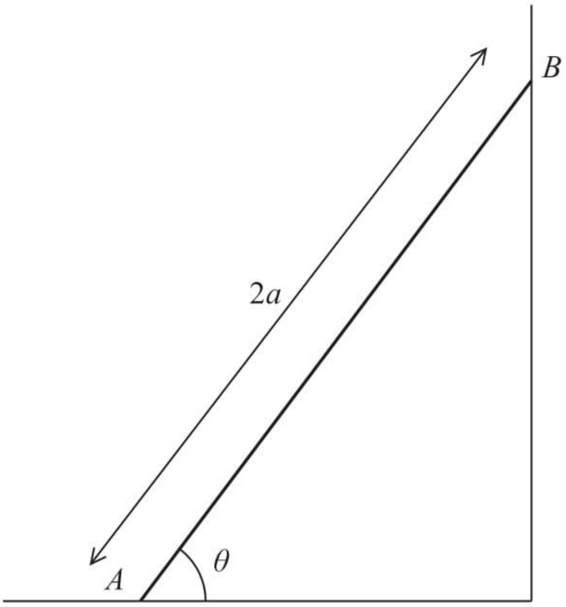


Figure 2

A beam AB has mass m and length $2a$.

The beam rests in equilibrium with A on rough horizontal ground and with B against a smooth vertical wall.

The beam is inclined to the horizontal at an angle θ, as shown in Figure 2.

The coefficient of friction between the beam and the ground is μ

The beam is modelled as a uniform rod resting in a vertical plane that is perpendicular to the wall.

Using the model,

(a) show that $\mu \geqslant \frac{1}{2}\cot\theta$

(5)

A horizontal force of magnitude kmg, where k is a constant, is now applied to the beam at A.

This force acts in a direction that is perpendicular to the wall and towards the wall.

Given that $\tan\theta = \frac{5}{4}$, $\mu = \frac{1}{2}$ and the beam is now in limiting equilibrium,

(b) use the model to find the value of k.

(5)

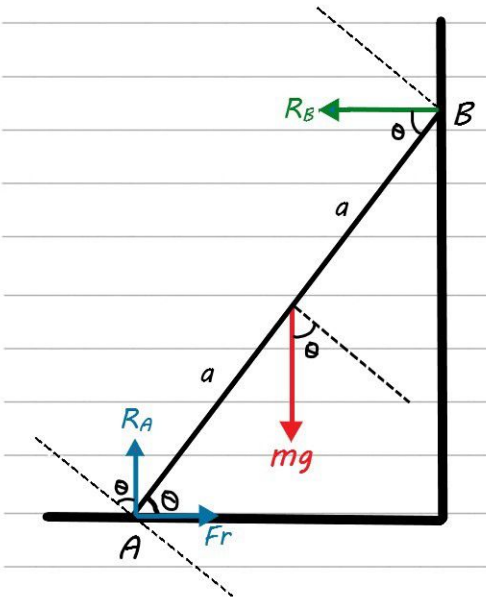
Question 3 continued

a) Firstly we need to highlight and note all important points in the question

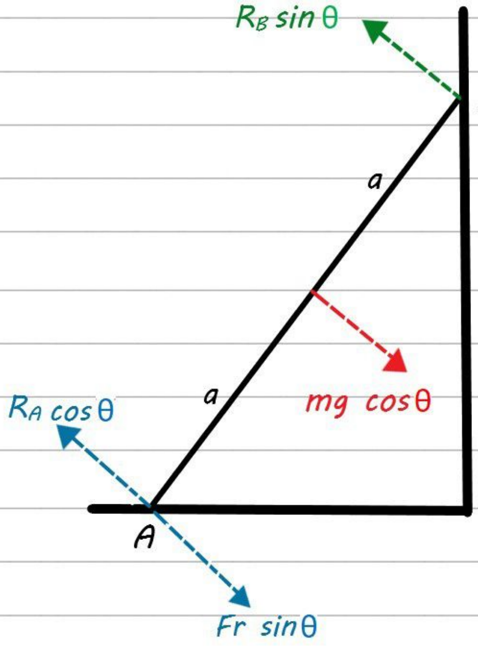
mass = m
length = 2a

rough surface => friction ≥ μR

uniform rod => weight acts downwards at centre of rod



(i)



(ii)

Looking at (i):

We know that the rod is at rest, and therefore is in static equilibrum

This means that forces acting upwards is equal to forces acting downwards

Question 3 continued

Resolving vertically in equilibrum,

$$R_A = mg$$

Looking at (ii):

The rod lies in equilibrum and there for the clockwise moments must be the same as anti-clockwise moments

Resolving moments around B,

$$(R_A \cos\theta)(2a) + (R_B \sin\theta)(0) = (Fr \sin\theta)(2a) + (mg \cos\theta)(a)$$

but, $R_A = mg$

$$\therefore \quad (mg \cos\theta)(2\cancel{a}) + 0 = (Fr \sin\theta)(2\cancel{a}) + (mg \cos\theta)(\cancel{a})$$

$$2mg \cos\theta = 2\ Fr \sin\theta + mg \cos\theta$$

$$mg \cos\theta = 2\ Fr \sin\theta$$

$$Fr = \frac{mg \cos\theta}{2 \sin\theta}$$

$$Fr = \frac{1}{2} mg \cot\theta \qquad [\because \cos/\sin = \cot]$$

We know that Friction $\leq \mu R_A$

$$\therefore \quad \frac{1}{2} mg \cot\theta \leq \mu R_A$$

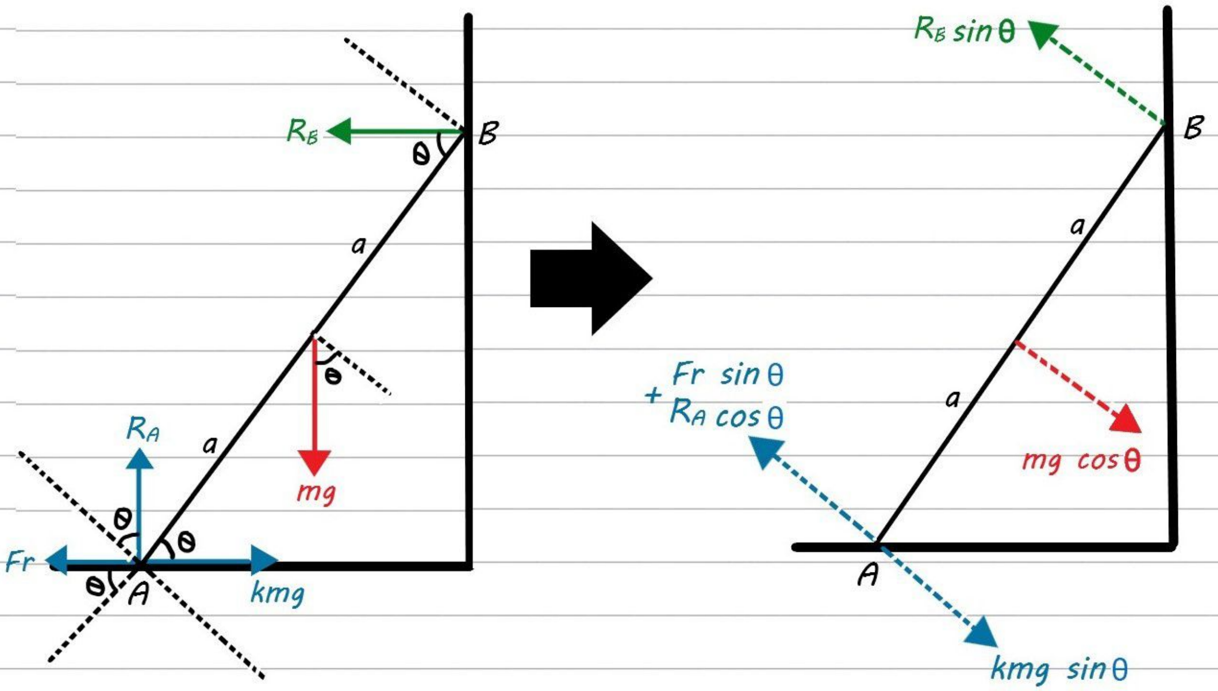
$$\frac{1}{2} mg \cot\theta \leq \mu mg \qquad [\because R_A = mg]$$

$$\frac{1}{2} \cot\theta \leq \mu$$

$$\mu \geq \frac{1}{2} \cot\theta \qquad Q.E.D.$$

Question 3 continued

b) Given, $\mu = \frac{1}{2}$ $\tan\theta = \frac{5}{4}$ => $\cot\theta = \frac{4}{5}$



The rod is in limiting equilibrium

$\therefore$ Friction $= \mu R_A = \frac{1}{2} mg$

Resolving forces around B:

$$(Fr \sin\theta)(2\cancel{a}) + (R_A \cos\theta)(2\cancel{a}) = (mg \cos\theta)(\cancel{a}) + (kmg \sin\theta)(2\cancel{a})$$

$$2\,Fr \sin\theta + 2mg\cos\theta = mg\cos\theta + 2kmg\sin\theta$$

$$mg\cos\theta = 2kmg\sin\theta - 2\,Fr\sin\theta$$

$$mg\cos\theta = (2\sin\theta)(kmg - Fr)$$

$$kmg - Fr = \frac{mg\cos\theta}{2\sin\theta}$$

$$kmg = \frac{1}{2}mg\cot\theta + Fr$$

$$k\cancel{mg} = \frac{1}{2}\cancel{mg}\cot\theta + \frac{1}{2}\cancel{mg}$$

Question 3 continued

$k = \frac{1}{2}\cot \quad + \frac{1}{2}$

$= \frac{1}{2} \times \frac{4}{5} + \frac{1}{2}$

$= \frac{2}{5} + \frac{1}{2}$

$k = \frac{9}{10}$

(Total for Question 3 is 10 marks)

4.

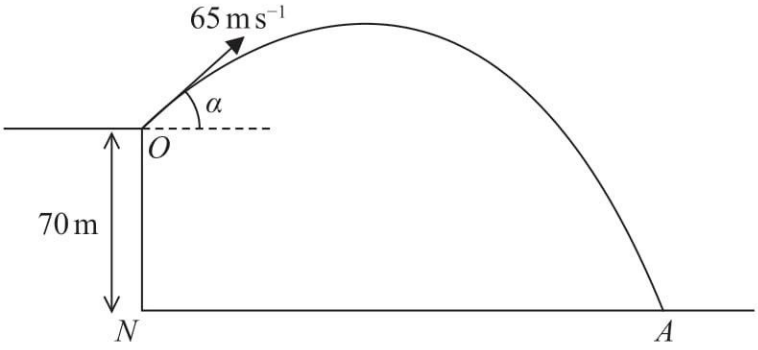


Figure 3

A small stone is projected with speed 65 m s^{-1} from a point O at the top of a vertical cliff.

Point O is 70 m vertically above the point N.

Point N is on horizontal ground.

The stone is projected at an angle α above the horizontal, where $\tan\alpha = \frac{5}{12}$

The stone hits the ground at the point A, as shown in Figure 3.

The stone is modelled as a particle moving freely under gravity.

The acceleration due to gravity is modelled as having magnitude 10 m s^{-2}

Using the model,

(a) find the time taken for the stone to travel from O to A,

(4)

(b) find the speed of the stone at the instant just before it hits the ground at A.

(5)

One limitation of the model is that it ignores air resistance.

(c) State one other limitation of the model that could affect the reliability of your answers.

(1)

Given,

initial speed $u = 65\ ms^{-1}$ $\xrightarrow{}$ vertical velocity = $65\sin a$; $\xrightarrow{}$ horizontal velocity = $65\cos a$

vertical displacement = ON = 70m

horizontal displacement = AN

vertical acceleration = $10\ ms^{-2}$

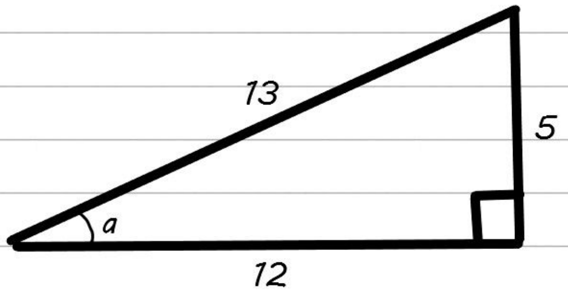
horizontal acceleration = 0 [This is because we ignore air resistance and thus the particle moves at a constant speed horizontally]

Question 4 continued

$\tan a = \frac{5}{12}$

$\therefore \quad \sin a = \frac{5}{13}$

$\therefore \quad \cos a = \frac{12}{13}$



a) To find the time of travel, we must see how long it takes for the particle to 70m vertically

∴ Resolving vertically using SUVAT:

$$\text{Vertical displacement} = ut + \frac{1}{2}at^2$$

$$70 = 65\sin a \times t + \frac{1}{2}(10)t^2$$

$$70 = 65 \times \frac{5}{13} \times t + 5t^2$$

$$70 = 25t + 5t^2$$

$$14 = 5t + t^2$$

$$t^2 + 5t - 14 = 0$$

$$t = 2 \quad \text{or} \quad t = -7$$

but time cannot be negative,

$\therefore \quad t = 2$ seconds

Question 4 continued

b) from part a, we found that the time taken to travel was 2 seconds

To find the speed after 2 seconds, we must resolve the velocities separately, horizontally and vertically

Resolving horizontally using SUVAT:

horizontal $v = u + at$

horizontal $v = 65\cos a + 0 \times 2$

$= 65 \times \frac{12}{13}$

$= 60\ ms^{-1}$

Resolving vertically using SUVAT:

vertical $v = u + at$

vertical $v = 65\sin a + 10 \times 2$

$= 65 \times \frac{5}{13} + 20$

$= 25 + 20$

$= 45\ ms^{-1}$

$\therefore$ velocity before hitting ground $= \begin{bmatrix} 60 \\ 45 \end{bmatrix} ms^{-1}$

Thus, to find the speed, we must find the magnitude of this vector

speed $= \sqrt{(60^2) + (45^2)}$

$= 75\ ms^{-1}$

Question 4 continued

c) Another limitation is that it ignores the size of the stone, i·e·
it is modelled as a particle, assuming gravity acts uniformly on it,
which may not be the case in reality

(Total for Question 4 is 10 marks)

5. At time t seconds, a particle P has velocity $\mathbf{v}\,\mathrm{m\,s^{-1}}$, where

$$\mathbf{v} = 3t^{\frac{1}{2}}\,\mathbf{i} - 2t\,\mathbf{j} \qquad t > 0$$

(a) Find the acceleration of P at time t seconds, where $t > 0$

(2)

(b) Find the value of t at the instant when P is moving in the direction of $\mathbf{i} - \mathbf{j}$

(3)

At time t seconds, where $t > 0$, the position vector of P, relative to a fixed origin O, is $\mathbf{r}$ metres.

When $t = 1$, $\mathbf{r} = -\mathbf{j}$

(c) Find an expression for $\mathbf{r}$ in terms of t.

(3)

(d) Find the exact distance of P from O at the instant when P is moving with speed $10\,\mathrm{m\,s^{-1}}$

(6)

$$\begin{tikzcd}\text{Displacement} \arrow[r, shift left, "\text{differentiate}"] & \text{Velocity} \arrow[l, shift left, "\text{integrate}"] \arrow[r, shift left, "\text{differentiate}"] & \text{Acceleration} \arrow[l, shift left, "\text{integrate}"]\end{tikzcd}$$

a) acceleration $= \frac{dv}{dt}$

$= (0{\cdot}5 \times 3t^{0{\cdot}5-1})\ i\ -\ 2\ j$

$a = (1{\cdot}5t^{-0{\cdot}5})\ i\ -\ 2\ j$

b) direction of $(i - j) = \tan^{-1}(\frac{-1}{1})$

when P is moving in direction of $(i - j)$, its direction is equal to direction of $(i - j)$

direction of P = direction of $(i - j)$

$\tan^{-1}(\frac{-2t}{3t^{0{\cdot}5}}) = \tan^{-1}(\frac{-1}{1})$

Question 5 continued

$$\left(\frac{-2t}{3t^{0.5}}\right) = -1$$

$$-2t = -3t^{0.5}$$

squaring both sides: $4t^2 = 9t$

$$4t^2 - 9t = 0$$

$$t(4t - 9) = 0$$

$$t = 0 \quad \text{or} \quad t = \frac{9}{4}$$

but $t > 0$

$\therefore$ $t = \frac{9}{4}$ seconds

c) displacement $r = \int v \cdot dt$

$$= \int 3t^{0.5}\, i - 2t\, j \cdot dt$$

$$= \frac{3t^{0.5+1}}{0.5+1} i - \frac{2t^2}{2} j + c$$

$$r = 2t^{1.5} i - t^2 j + c$$

given, at $t = 1$, $r = -j$

$$-j = 2(1)^{1.5} i - (1)^2 j + c$$

$$-j = 2i - j + c$$

$$c = -2i$$

$\therefore$ $r = 2t^{1.5} i - t^2 j - 2i$

$$= (2t^{1.5} - 2)\, i - t^2 j$$

d) the speed of a body is the magnitude of its velocity

given, speed = 10 ms^{-1}

magnitude of velocity = 10ms^{-1}

$|v| = 10\ ms^{-1}$

$$\sqrt{(3t^{0\cdot5})^2 + (-2t)^2} = 10$$

squaring both sides: $(3t^{0\cdot5})^2 + (-2t)^2 = 10^2$

$$9t + 4t^2 = 100$$

$$4t^2 + 9t - 100 = 0$$

$t = 4$ or $t = -6\cdot25$

but $t > 0$,

$\therefore$ $t = 4$ seconds

at $t = 4$, the particle is moving at speed 10ms^{-1}

substituting $t = 4$ in equation for **r**:

$$r = [2(4)^{1\cdot5} - 2]\ i - (4)^2 j$$

$$= (2 \times 8 - 2)i - 16\mathbf{j}$$

$$= 14i - 16\mathbf{j}$$

distance = magnitude of r

$$= \sqrt{(14)^2 + (-16)^2}$$

$$= \sqrt{196 + 256}$$

$$= \sqrt{452}$$

$= 4\sqrt{113}$ metres

Question 5 continued

Turn over ▶

Question 5 continued

(Total for Question 5 is 14 marks)

TOTAL FOR MECHANICS IS 50 MARKS